Lecture 9 Summary Tues 12/08/14

Vocabulary

* convergent function $\vec{x}$ (ie. sequence)
* equivalent metrics

Examples

(None)

Homework

- Show that $\vec{x}: \mathbb{Z}_{>0} \to X$ satisfies (*) iff $\lim_{n\to\infty} d(x_n, x) = 0$.

  (*) "Let (X, d) be a metric space. Let $x \in X$.
  A function $\vec{x}: \mathbb{Z}_{>0} \to X$, $n \mapsto x_n$ **converges** to x if $\vec{x}$ satisfies:
  if $\varepsilon \in \mathbb{R}_{>0}$ then there exists $N \in \mathbb{Z}_{>0}$ such that
  if $n \in \mathbb{Z}_{>0}$ and $n > N$ then $d(x_n, x) < \varepsilon$"

- If $\vec{x}: \mathbb{Z}_{>0} \to X$, $n \mapsto x_n$ is a sequence in X and $x, y \in X$ and

  $\lim_{n\to\infty} x_n = x$ and $\lim_{n\to\infty} x_n = y$ then $x = y$.

- Show that if [metrics on X] d_1 and d_2 satisfy (**) then d_1 and d_2 are equivalent metrics.

  (**) "If $x, y \in X$ then there exists $c_1 \in \mathbb{R}_{>0}$ and $c_2 \in \mathbb{R}_{>0}$ such that
  $d_1(x, y) \leq c_1 d_2(x, y)$ and $d_2(x, y) \leq c_2 d_1(x, y)$"

- Do the notes (Lemma 2.12) use (**) or do they use:

  (**)' "There exist $c_1 \in \mathbb{R}_{>0}$ and $c_2 \in \mathbb{R}_{>0}$ such that
  if $x, y \in X$ then $d_1(x, y) \leq c_1 d_2(x, y)$ and $d_2(x, y) \leq c_2 d_1(x, y)$"

- Why isn't it an "if and only if" statement?

- Let (X, d) be a metric space. X is a topological space with the metric space topology. Let $A \subseteq X$ and let $\bar{A}$ be the closure of A. Show that

  $$\bar{A} = \{x \in X \mid \text{there exists } \vec{a}: \mathbb{Z}_{>0} \to A,\ n \mapsto a_n \text{ with } \lim_{n\to\infty} a_n = x\}$$